3D Gravity

martes, 6 de agosto de 2024 11:10

Lower-D gravity?

$$\Phi_N \sim -\frac{GM}{r^{D-3}}$$

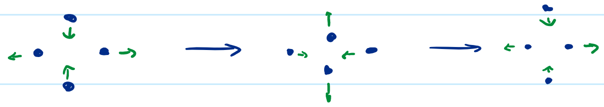
$D < 4$

Gravity is weaker at small r: improved UV behavior
good for QFT fluctuations

not decaying at large r: very sensitive to
long distance
May lose black holes!

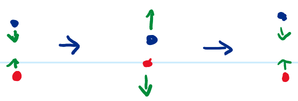
- Gravitational d.o.f.'s: more polarizations in $D > 4$

No propagating d.o.f's already in $D=3$:
no room for quadrupolar (shear) motion!



: impossible in $D < 4$

[gauge fields can oscillate in $D=3$
not in $D=2$



]

[scalars in all $D \geq 2$]

Scales and dimensions

In $D=4$ (with $c=1$) GM is a length

In arbitrary D, $(GM)^{1/D-3}$ is a length

In $D=3$ GM is dimensionless

In $D=3$ GM is dimensionless

The mass does not determine a length scale, Thus There can't be a black hole horizon solely determined by M. A particle coupled To gravity creates a conical defect (dimensionless).

[Mass in $D \geq 4$ is measured from "extrinsic curvature defect" of large-radius spheres
M in $D=3$ it's measured from conical angle defect of large-radius circles]

- Need a length scale To have a black hole.

Eg a cosmological radius L, from a cosmological constant $\Lambda \sim 1/L^2$

But having a length scale is necessary, not sufficient: we also need attraction.

Mass in 3D doesn't attract by itself: conical defect is global effect

$\Lambda > 0$ won't do: expansion

$\Lambda < 0$ may do: collapse

So we'll consider $\Lambda \sim -1/L^2$. The black holes will have size $\propto L$ and Thus will be "of AdS size", ie "large AdS bhs". $\nexists$ small AdS bhs in $D=3$

- Gravity in 3D has no propagating dof's

3D Weyl = 0 identically [in 3D Weyl=0 $\not\Rightarrow$ conf flat
CoTTon=0 $\Leftrightarrow$ conf flat]

Riemann determined by Ricci.
BuT Ricci is determined by EinsTein equations: once a 3D spacetime is required To satisfy The EinsTein eqs, There doesn'T remain any freedom in iT (no grav waves).

With cosmological constanT we'll have

$$R_{ij} = -\frac{2}{l^2} g_{ij} \qquad \text{ie Ricci = constant}$$

so Then <u>Riemann = constanT</u>. If $\Lambda < 0$

$\Rightarrow$ <u>All solutions are locally equivalenT To AdS_3</u>

[3D graviTy is Topological, only boundary dof's]

Can only differ:

(i) in global sTructure (like plane and cylinder differ)

(ii) in distributional singularities (like plane and cone differ)
Sources: conical points
line sources

So we Know The local geometry: AdS_3
and we're going To see how To find
<u>geometries That differ globally.</u>

<u>AdS_3</u>:

BTZ '92
BHTZ '93

We know S^3 is described as a surface in $\mathbb{R}^4$

$x_0^2 + x_1^2 + x_2^2 + x_3^2 = 1$ in $ds^2 = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2$

(unit radius $L=1$)

In order to change $S^3 \to AdS_3$ we flip two signs: to go Lorentzian (have time) and to have -ve curvature

$$ds^2 = -dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2$$

$$-x_0^2 - x_1^2 + x_2^2 + x_3^2 = -1$$

Can solve for surface

$$\underbrace{-x_0^2 - x_1^2}_{\substack{\text{polar} \\ (t,r)}} + \underbrace{x_2^2 + x_3^2}_{\substack{\text{polar} \\ (r,\phi)}} = -1$$

$x_2 = r \sin\phi$

$x_3 = r \cos\phi$

$x_0 = \sqrt{r^2+1} \sin t$: periodic t?

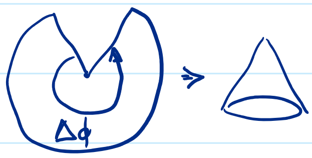
$x_1 = \sqrt{r^2+1} \cos t$

No: "unwrap" to global covering space $-\infty < t < \infty$

(but there remains a memory of this periodicity)

$$\Rightarrow ds^2 = -(r^2+1)\,dt^2 + \frac{dr^2}{r^2+1} + r^2 d\phi^2$$

Take $\phi \sim \phi + \Delta\phi$



If $\Delta\phi \neq 2\pi$: conical singularity at $r=0$

Naked: No horizon

$-g_{tt} > 0 \quad \forall r$

BTZ:

Now solve the surface equation by pairing up the coordinates in a different way, in boost-like fashion:

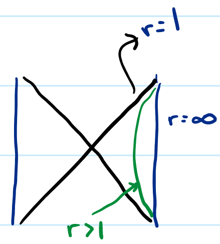
$$\underbrace{-x_0^2 - \overbrace{x_1^2 + x_2^2}^{\text{boost}}}_{\text{boost}} + x_3^2 = -1$$

$$x_1 = r\cosh\phi \qquad -x_1^2 + x_3^2 = -r^2$$
$$x_3 = r\sinh\phi$$

$$x_0 = \sqrt{r^2-1}\,\sinh t \qquad -x_0^2 + x_2^2 = r^2 - 1$$
$$x_2 = \sqrt{r^2-1}\,\cosh t$$

$$\Rightarrow \quad ds^2 = -(r^2-1)\,dt^2 + \frac{dr^2}{r^2-1} + r^2 d\phi^2$$

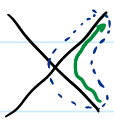
but now in principle $-\infty < t < \infty$
$-\infty < \phi < \infty$



This is Rindler-AdS_3

Horizon at $r=1$ is non-compact acceleration horizon.

[This is the AdS version of Rindler space:



$r = 1 + \xi^2/2 \quad \xi^2 \ll 1 \Rightarrow ds^2 \simeq -\xi^2 dt^2 + d\xi^2 + d\phi^2$: 3D Rindler

$= -dT^2 + dX^2 \qquad T = \xi\sinh t$
$X = \xi\cosh t$]

[∂_t and ∂_ϕ are boost vectors

$\partial_t = x_2 \dfrac{\partial}{\partial x_0} + x_0 \dfrac{\partial}{\partial x_2}$ Timelike for $-x_0^2 + x_2^2 > 0$ $\quad r^2 - 1 > 0$

$$\partial_t = x_2 \frac{\partial}{\partial x_0} + x_0 \frac{\partial}{\partial x_2} \qquad \text{timelike for} \quad -x_0^2 + x_2^2 > 0 \qquad r^2 - 1 > 0$$

$$\partial_\phi = x_1 \frac{\partial}{\partial x_3} + x_3 \frac{\partial}{\partial x_1} \qquad '' \quad '' \quad -x_1^2 + x_3^2 > 0 \qquad \text{“} r^2 < 0 \text{”}$$

$$\text{spacelike} \quad '' \quad -x_1^2 + x_3^2 < 0 \qquad r^2 > 0$$

Rindler-AdS_3 is just a patch of AdS_3.

We now introduce a global difference with AdS_3:

<u>Identify along orbits of ∂_ϕ</u>

$$\phi \sim \phi + \Delta\phi \qquad (\text{ok where } r^2 > 0)$$

$\Delta\phi$ is arbitrary: need not be 2π since ϕ is a boost coordinate, and $r=0$ is hidden behind the horizon (see below for more on $r=0$)

Horizon now is <u>compact</u>: black hole horizon

$$ds^2 = -(r^2-1)\,dt^2 + \frac{dr^2}{r^2-1} + r^2 d\phi^2 \qquad \text{with } \phi \sim \phi + \Delta\phi$$

is the <u>BTZ black hole</u> with horizon at $r=1$, "area" (length) $A_H = \Delta\phi$

The form above is not the "canonical metric".

Rescale $\phi = \frac{\Delta\phi}{2\pi}\tilde{\phi}$ so $\Delta\tilde{\phi} = 2\pi$

and absorb factors in $r = \frac{2\pi}{\Delta\phi}\tilde{r}$

$$t = \frac{\Delta\phi}{2\pi}\tilde{t}$$

Then

$$ds^2 = -(\tilde{r}^2 - 8GM)\,d\tilde{t}^2 + \frac{d\tilde{r}^2}{\tilde{r}^2 - 8GM} + \tilde{r}^2 d\tilde{\phi}^2$$

with $8GM = \left(\frac{\Delta\phi}{2\pi}\right)^2$ $\qquad \tilde{\phi} \sim \tilde{\phi} + 2\pi$

M is the black hole mass (from detailed analysis)

$$M = \frac{(D-2)\,\Omega_{D-2}}{16\pi G}\,\mu \underset{D=3}{=} \frac{\mu}{8G}$$

Notice global AdS_3 has $M = -1/8G$

BHs have $M > 0$.

$-1/8G < M < 0$ are conical singularities (particles)

["M=0 black hole" is mildly singular. It is obtained by identifying along a null boost.]

[$r=0$: null hypersurface : $\partial/\partial\phi$ is null there

no curvature singularity

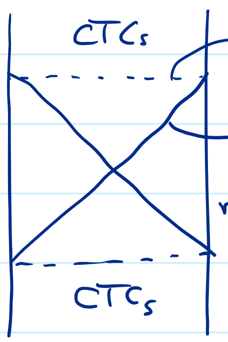
metric can be analytically continued

There are CTCs beyond: points identified along timelike directions (boost)

some null geodesics are incomplete

(similar to Lorentzian Taub-NUT and Misner space)

Keep only $r > 0$]



$r=0$ but it is actually null, in the ϕ direction $|\partial/\partial\phi|^2 = 0$

$r = r_H$

$r = \infty$

So:

$$\boxed{\text{AdS}_3 \text{ with points identified along orbits of a boost vector is the BTZ black hole}}$$

- Global definition, indep of coordinates!
- Mass is given by period of identification

Other forms of the metric:

from $ds^2 = -(r^2 \pm 1)\, dt^2 + \frac{dr^2}{r^2 \pm 1} + r^2 d\phi^2$ AdS$_3$ / BTZ

proper radius $d\rho^2 = \frac{dr^2}{r^2 \pm 1}$

$+: \; r = \sinh\rho \qquad ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\phi^2$ AdS$_3$

no zero, no horizon

zero at $\rho = 0$: origin of rotation

$-: \; r = \cosh\rho \qquad ds^2 = -\sinh^2\rho\, dt^2 + d\rho^2 + \cosh^2\rho\, d\phi^2$ BTZ

zero at $\rho = 0$, horizon

no zero, finite area horizon

[Observe Euclidean $t \to i\tau$ are equivalent $\tau \Leftrightarrow \phi$

Euclidean thermal AdS$_3$ and BTZ only differ in the choice of Euclidean time]

Rotation: identify ϕ after a shift in time (equivalently, along a vector $\partial/\partial\phi + \alpha\, \partial/\partial t$, $|\alpha|<1$)

In this case there are no incomplete geodesics. Excise by hand region w/ CTCs

Easy to find that $\qquad G = 1/8$

$$ds^2 = -\left(r^2 - M + \frac{J^2}{4r^2}\right) dt^2 + \frac{dr^2}{r^2 - M + J^2/4r^2} + r^2\left(d\phi - \frac{J}{2r^2}\, dt\right)^2$$

$$= -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}\, dt^2 + \frac{r^2\, dr^2}{(r^2 - r_+^2)(r^2 - r_-^2)} + r^2\left(d\phi - \frac{r_+ r_-}{r^2}\, dt\right)^2$$

$$r_+^2 r_-^2 = \frac{J^2}{4}$$

$$r_+^2 + r_-^2 = M$$

Usual pattern: $\quad g_{t\phi} \propto J$

$$-8GM + \frac{J^2}{4r^2}$$

$\hookrightarrow$ centrifugal repulsion

$\exists$ extremal limit $r_+ = r_- \quad M = |J|$

Regular w/ finite area

This is also obtained from identifications along a null boost

The BTZ black hole plays a central role in one of the best understood and most detailedly studied instances of holography: AdS_3/CFT_2

of the best understood and most detailedly studied instances of holography: AdS_3/CFT_2

Both sides of the duality are eminently solvable due to the large amount of symmetry, and allows precise computations and checks.

BTZ also underlies many precise calculations of the Bekenstein-Hawking entropy in string theory, including the first one by Strominger + Vafa.

BTZ black hole entropy from CFT_2

$$ds^2 = -\left(\frac{r^2}{\ell^2} - 8GM\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell^2} - 8GM} + r^2\, d\phi^2 \qquad (\text{remove} \sim)$$

$\ell =$ AdS_3 radius

$$r_H = \ell\sqrt{8GM}$$

$$A_H = 2\pi r_H = 2\pi \ell \sqrt{8GM}$$

$$S = \frac{A_H}{4G} = \frac{\pi \ell}{2}\sqrt{\frac{8M}{G}} = \pi \ell \sqrt{\frac{2M}{G}}$$

We want to reproduce this as the degeneracy of a CFT_2 with central charge $c = \frac{3\ell}{2G}$ at high energy / temperature.

There exists a general formula for the degeneracy of states e^S of a CFT_2

degeneracy of states e^{S} of a CFT_2

"Cardy formula"

$L =$ length of spatial circle of CFT_2

$E =$ Total energy

$$S_{CFT} = 2\pi \sqrt{\frac{cE}{3}\frac{L}{2\pi}}$$

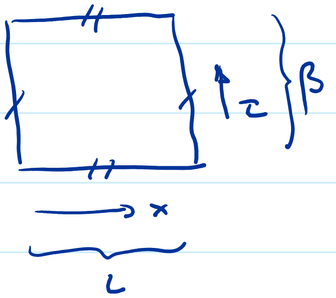
Now plug in here $\begin{cases} E = M \\ L = 2\pi \ell \end{cases}$ $\quad c = \dfrac{3\ell}{2G}$

$$S_{CFT} = 2\pi \sqrt{\frac{M\ell^2}{2G}} = \pi \ell \sqrt{\frac{2M}{G}} = S_{BH}$$

$$\boxed{S_{CFT} = S_{BH}}$$: it works exactly!

Cardy formula for the degeneracy of CFT_2 states at high energy / Temperature

$Z(\beta) = \mathrm{Tr}\, e^{-\beta H}$: compute it from Euclidean path integral on Torus



$\tau \sim \tau + \beta$

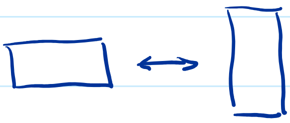
$x \sim x + L$

$Z(\beta, L)$ is dimensionless, and the only scales are

$Z(\beta,R)$ is dimensionless, and the only scales are β and L, so it must be that

$$Z(\beta,L) = Z(\beta/L) \qquad \left(= Z(\beta/L, 1) \right)$$

Now we use <u>modular invariance</u>, which is a property of CFT_2 at one loop, and which implies that $Z(\beta,L) = Z(L,\beta)$



$$= Z\left(\frac{L^2}{\beta}, L\right)$$

Then for fixed L we can write

$$Z(\beta) = Z\left(\frac{L^2}{\beta}\right)$$

(often one writes $L = 2\pi R$ and then $Z(\beta) = Z\left(\frac{4\pi^2 R^2}{\beta}\right)$)

This relates the density of states at high-Temp $\beta \to 0$, to the density at low Temp, $\beta \to \infty$

At low temperatures, Z is dominated by the vacuum state of the CFT_2 on a circle, with energy

$$E_0 = -\frac{c}{12}\frac{2\pi}{L} = -\frac{\pi c}{6L}$$

(Casimir energy on a circle)

so

$$Z(L^2/\beta) = Z(\tilde{\beta}) \approx e^{-\tilde{\beta} E_0} \qquad \tilde{\beta} = L^2/\beta$$

$$= e^{\frac{L^2}{\beta}\frac{\pi c}{6L}}$$

$$= e^{\frac{\pi}{\beta}\frac{c}{6L}}$$

ie for $\beta \to 0$ $\quad Z(\beta) \approx e^{\frac{\pi L c}{6\beta}} = e^{-\beta F}$

$$\Rightarrow \quad F = -\frac{\pi L}{6} c T^2 \qquad T = 1/\beta$$

Using standard Thermodynamics we find That

$$\begin{cases} F = E - TS \\ dF = -S\,dT \\ dE = T\,dS \end{cases}$$

$$\left.\begin{aligned} E &= \frac{\pi L}{6} c T^2 \\ S &= \frac{\pi L}{3} c T \end{aligned}\right\} \Rightarrow \boxed{S = 2\pi \sqrt{\frac{cE}{3} \frac{L}{2\pi}}}$$

2D Gravity

martes, 6 de agosto de 2024 11:10

In $D=3$ we had Weyl$=0$ so all solutions of Einstein's eqns $R_{ij} = \Lambda g_{ij}$ $(\Lambda \gtreqless 0)$ were locally equivalent to AdS_3, $Mink_3$, dS_3 and only differed in global properties.

In $D=2$ we have $R_{ij} = \frac{1}{2} g_{ij} R$ identically, so we can't find any solutions since there are no equations!

The reason is that $I_{EH} = \frac{1}{16\pi G} \int d^2x \sqrt{-g}\, R$ is a topological invariant (Euler number) and its variation only yields bdry terms

In order to have non-trivial equations (although not necessarily local propagating d.o.f.'s) we can introduce a scalar field (dilaton):

$$I = \frac{1}{16\pi G} \int d^2x \sqrt{-g}\, \Phi(x) R + \dots$$

(review Grumiller et al '02)

(in $D>2$ Φ can be absorbed in a change of conformal frame but not in $D=2$)

Observe that $\Phi(x)$ acts to give an x-dependent gravitational coupling $\frac{1}{G_{eff}} = \frac{\Phi(x)}{G}$

If we have a 2D BH, its horizon is a point. We expect its entropy to be

$$S = \frac{1}{4G_{eff}} = \frac{\Phi(x_H)}{4G}$$

This can (and should) be derived from a proper analysis of the complete Euclidean action, but it is generally valid if there are no more curvatures in the action.

It is also well motivated when the 2D bh is viewed as the result of a spherical reduction of a higher-D black hole.

Eg

$$ds^2 = \underbrace{-f(r)\,dt^2 + \frac{dr^2}{f(r)}}_{2D\ bh} + \underbrace{r^2}_{\Phi(t,r)} d\Omega_2$$

If we only consider spherically symmetric configurations, we can integrate the angles (θ, ϕ) in the action and obtain an effective 2D gravity theory of the form above.

Theory of the form above.

We see that $\Phi(t,r)$ measures the area of the S^2, so $A(r_H) = 4\pi\,\Phi(r_H)$

and the formula above gives the correct entropy

(The factor 4π appears from the integration of $\int \sin\theta\, d\theta\, d\phi$ in the action).

2D dilaton theories differ in the choice of terms "..." in the action.

We'll only consider two simple but important cases:

JT gravity

Jackiw '83

Teitelboim '83

$$I = \frac{1}{16\pi G}\int d^2x\,\sqrt{-g}\,\Phi\,(R - \Lambda) \qquad \text{with } \Lambda = -\frac{2}{L^2} = -2$$

$L=1$

2D effective string theory ("CGHS")

$$I = \frac{1}{16\pi G}\int d^2x\,\sqrt{-g}\,(\Phi R - \Lambda) \qquad \text{with } \Lambda = -4\lambda^2 \text{ (conventionally)}$$

This is not the most usual way of presenting it, rather one performs a Weyl transformation to the "string metric"

$$g_{ij} = e^{-2\phi} \hat{g}_{ij} \quad \text{with} \quad \Phi = e^{-2\phi}$$

and then (verify!), removing hats from $\hat{g}_{ij}$

$$I = \frac{1}{16\pi G} \int d^2x \sqrt{-g}\, e^{-2\phi} \left(R + 4(\nabla\phi)^2 + 4\lambda^2 \right)$$

"ghost" sign? no problem since there are no propagating dofs : "kinetic term" $(\nabla\phi)^2$ can always be removed w/ a conformal change of frame as we have seen

These two theories are important because they admit classical black hole solutions, but also because, when coupled to 2D conformal field matter, the quantum CFT can be solved and its backreaction on the black hole can be computed.

In particular, JT gravity has played a central role in recent computations of the "Page curve" for evaporating black holes – a test of unitary evolution. It has also attracted a lot of attention as a highly solvable model of holographic quantum gravity, since its dynamics describes the low-energy physics of the SYK model, a quantum-mechanical theory that captures many highly non-trivial properties of the statistical mechanics of black hole microscopics.

They also appear as the spherical reduction of relevant higher-D black holes

JT bh from
- BTZ
- near-extremal charged bhs

CGHS bh from
- near-extremal dilaton magnetic bhs
- near-extremal NS5 branes
- $D \to \infty$ neutral black holes

(also from BTZ but trickier)

Black holes in JT gravity

martes, 6 de agosto de 2024 11:10

$$I = \frac{1}{16\pi G}\int_M d^2x\sqrt{-g}\,\Phi(x)\,(R+2) + \frac{1}{8\pi G}\int_{\partial M} dx\sqrt{|h|}\,\Phi\,(\underbrace{K}_{\text{GHY}}\underbrace{-1}_{\text{counterterm}})$$

GHY boundary terms are important when are computes the action, when quantum effects are introduced, or when a dynamical boundary cutoff is introduced ("near-AdS" "Schwarzian particle")

Can add a Topological term (Euclideanized)

$$I_{Topo} = -S_0\left(\frac{1}{16\pi G}\int_M d^2x\sqrt{g}\,R + \frac{1}{8\pi G}\int_{\partial M} dx\sqrt{|h|}\,K\right) = -S_0\,\chi$$

χ: Euler characteristic

$\chi = 2(1-g)-b$

This gives a constant contribution to the action and to the entropy.

When $S_0 \gg 1$ it suppresses the contributions from higher topologies: $e^{-I_{Topo}} = e^{S_0\chi} : e^{2S_0}(1, e^{-2S_0}, e^{-4S_0}, \dots)$

sphere Torus

Vary $\delta\Phi \Rightarrow R=-2$: in 2D, R determines all the curvature, so all the solutions must be <u>locally equivalent to AdS_2</u>

- <u>No dynamics in AdS_2</u>

• No dynamics in AdS_2

If we put an excitation in $AdS_{D\geq 4}$, its effect at large distances dies away as $\frac{1}{r^{D-3}}$: can have finite energy spectrum of excitations.

In $D=2$ the lines of force don't dilute since there are no directions in which to expand

$\Rightarrow$ any excitation will have a large effect at asymptotic infinity : "non-normalizable"

= no finite-energy excitations in AdS_2

We can have finite dynamics ("normalizable excitations") if we introduce a cutoff near the AdS_2 boundary, thus regularizing the infinite energy : near-AdS_2 boundary dynamics.

• This has a counterpart in the properties of a putative CFT_1 dual:

a CFT has $T^i_i = 0$

In 1+0 dimensions this means $T^t_t = 0$: zero energy!

The theory can only describe ground states, which is consistent (eg BPS ground states) but which is consistent (eg BPS ground states) but not very interesting for bh evaporation.

Moreover, consider the possible form of the density of states $\rho(E) = \frac{dn}{dE}$

In 1+0. The only scale is E (in higher D there is the spatial volume) so it must be

$$\rho(E) = A\,\delta(E) + \frac{B}{E}$$

$A\,\delta(E)$: ground-state degeneracy

$\frac{B}{E}$: divergent in infrared $E \to 0$

must introduce an IR cutoff in order to have finite energy states.

The cutoff breaks conformal symmetry

Near-conformal theory

This is the dual counterpart of near-AdS_2 dynamics

AdS_2 geometries and JT solutions

Consider, as we did in 3D, the hyperboloid embedding of AdS_2:

$$ds^2 = -dx_0^2 - dx_1^2 + dx_2^2 \qquad -x_0^2 - x_1^2 + x_2^2 = -1$$

Depending on what we choose to be the time

Depending on what we choose to be the time coordinate, we'll get different patches of AdS_2.

Take t a polar angle in the "time plane" (x_0, x_1)

$$\left.\begin{aligned} x_0 &= \cosh\rho \sin t \\ x_1 &= \cosh\rho \cos t \\ x_2 &= \sinh\rho \end{aligned}\right\}$$

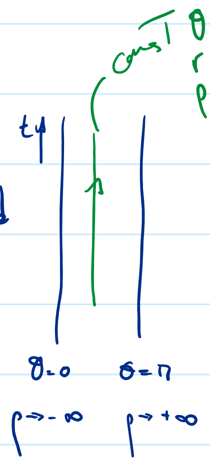
Global AdS_2

or

$$\left.\begin{aligned} x_0 &= \sqrt{r^2+1}\ \sin t \\ x_1 &= \sqrt{r^2+1}\ \cos t \\ x_2 &= r \end{aligned}\right\}$$

	Boundary	Center
$ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2$	$\rho \to \pm\infty$	$\rho = 0$
$r = \sinh\rho$		
$= -(r^2+1)\, dt^2 + \dfrac{dr^2}{r^2+1}$	$r \to \pm\infty$	$r = 0$
$r = \cot\theta$		
$= \dfrac{-dt^2 + d\theta^2}{\sin^2\theta}$	$\theta = 0, \pi$	$\theta = \pi/2$
$t = u+v$, $\theta = u-v$		
$= -4\dfrac{du\, dv}{\sin^2(u-v)}$	$u-v = 0, \pi$	$u-v = \pi/2$

There are no horizons, $-g_{tt} > 0$ everywhere.
The two asymptotic boundaries are disconnected
(This is then like a 1+1 wormhole)



Now take t to be a boost coordinate

$\rho \to -\infty \quad \rho \to +\infty$

Now Take t To be a boost coordinate

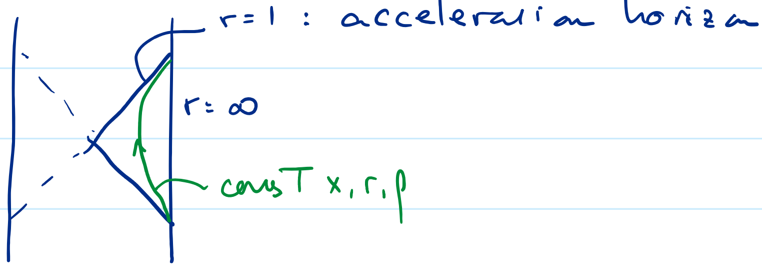
$$\left.\begin{aligned} X_0 &= \sinh\rho \sinh t \\ X_1 &= \cosh\rho \\ X_2 &= \sinh\rho \cosh t \end{aligned}\right\} \text{Thermal (Rindler) } AdS_2$$

or

$$\left.\begin{aligned} X_0 &= \sqrt{r^2-1} \sinh t \\ X_1 &= r \\ X_2 &= \sqrt{r^2-1} \cosh t \end{aligned}\right\}$$

We'll have a Rindler horizon where $X_2^2 - X_0^2 = 0$

	Bdry	Horizon
$ds^2 = -\sinh^2\rho \, dt^2 + d\rho^2$ $r = \cosh\rho$	$\rho \to \infty$	$\rho = 0$
$= -(r^2-1)\, dt^2 + \dfrac{dr^2}{r^2-1}$ $r = \coth x$	$r \to \infty$	$r = 1$
$= \dfrac{-dt^2 + dx^2}{\sinh^2 x}$ $t = u + v$ $x = u - v$	$x \to 0$	$x \to \infty$
$= -4 \dfrac{du\, dv}{\sinh^2(u-v)}$	$u - v \to 0$	$u - v \to \infty$



Like in BTZ, we can rescale

$r \to r/r_H \qquad t \to r_H t$

To find
$$ds^2 = -(r^2 - r_H^2)\, dt^2 + \frac{dr^2}{r^2 - r_H^2} \qquad r_H = \frac{2\pi}{\beta}$$

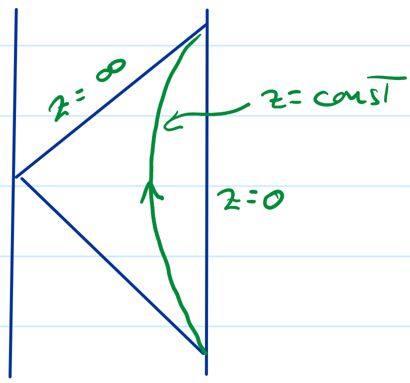
But, unlike in BTZ, here we cannot make global identifications that make them different!
(There is no angular direction to identify)

A final form is AdS$_2$ in Poincaré coordinates:

$$X_0 = \frac{1}{2z}(1 - t^2 + z^2) \qquad -X_0^2 - X_1^2 + X_2^2 = -1$$

$$X_1 = -t/z$$

$$X_2 = \frac{1}{2z}(1 + t^2 - z^2)$$



$$ds^2 = \frac{-dt^2 + dz^2}{z^2} \qquad t = u+v$$
$$z = u - v$$

$$= -4\frac{du\,dv}{(u-v)^2}$$

$$= -r^2 dt^2 + \frac{dr^2}{r^2} \qquad z = 1/r$$

when $r \gg 1$, the other forms of AdS$_2$ reduce to this one.

All of these are locally and globally AdS$_2$
What makes the difference is the dilaton.
Vary δg^{ij} (The equation is $T^{\Phi}_{ij} = 0$)

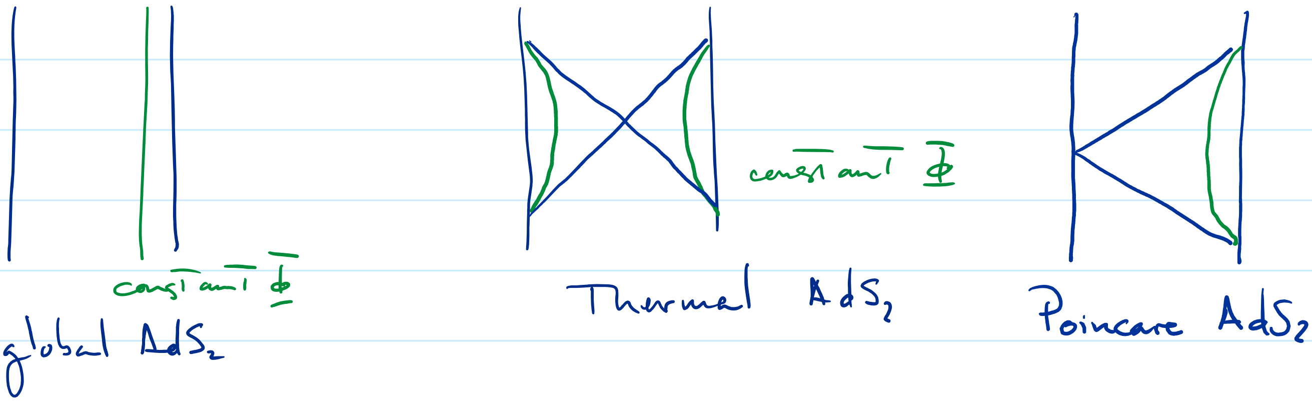
$$\Rightarrow \quad \frac{1}{2} g_{ij} \Box \Phi = \nabla_i \nabla_j \Phi = g_{ij} \Phi$$

For $ds^2 = -(r^2+\kappa)\,dt^2 + \frac{dr^2}{r^2+\kappa}$ $\qquad \kappa = \pm 1, 0$

The dilaton eqns are solved for all κ by

$$\Phi = a r \qquad (a: \text{constant})$$

but notice that r is different in each case:



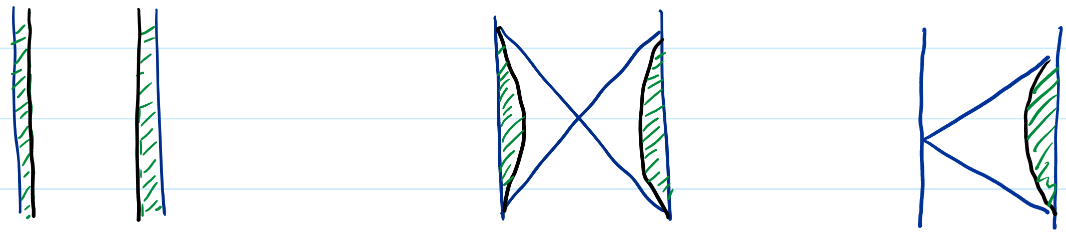
$\Phi \to \infty$ at the boundary $r \to \infty$

Near-AdS_2 and Schwarzian dynamics

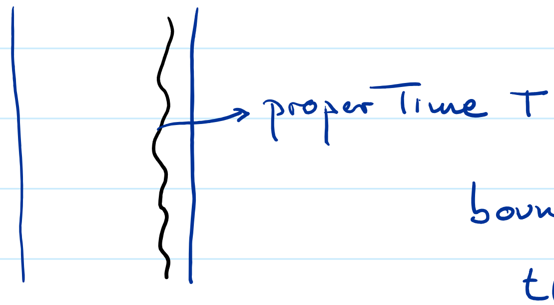
Introduce a cutoff at $\Phi = \text{const} \gg 1$
$\qquad\qquad r = 1/\varepsilon$

"renormalized"
$$\phi\big|_{\text{cutoff}} = \phi_b / \varepsilon$$

This gives near-AdS_2 geometries which are inequivalent and which can have a dynamical boundary:



The dynamics of near-AdS_2 JT gravity is the dynamics of a "particle" (or 2) moving near the boundary of AdS_2:



boundary Trajectory
$t(T)\,,\; z(T)$

$$ds^2\Big|_\partial = -\frac{dT^2}{\varepsilon^2} = \frac{-dt^2 + dz^2}{z^2} = \frac{-t'^2 + z'^2}{z(T)^2}\, dT^2 \qquad {}' = \frac{d}{dT}$$

$$t'^2 - z'^2 = \frac{z^2}{\varepsilon^2} \qquad z(T) = \varepsilon\, t'(T) + O(\varepsilon^2)$$

Compute extrinsic curvature of boundary

$$K = 1 + \varepsilon^2 \{t, T\} + O(\varepsilon^4)$$

$$\{t, T\} \equiv \frac{t'''}{t'} - \frac{3}{2}\frac{t''^2}{t'^2} \qquad \text{Schwarzian derivative}$$

Symmetric under $t \to \dfrac{at+b}{ct+d}$ $\quad SL(2,\mathbb{R})$

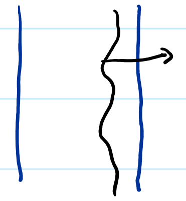
$$\left[\text{Then } \; t(T) = \frac{aT+b}{cT+d} \iff \{t,T\} = 0\right]$$

Gravitational action: bulk term vanishes

$$I_{JT}\Big|_{\text{on-shell}} = \frac{1}{8\pi G}\int_{\partial M} dT \sqrt{|h|}\, \Phi\,(K-1)$$

$$= \frac{\phi_b}{8\pi G}\int_{\partial M} dT\, \{t, T\}$$: Schwarzian action for the boundary particle

$t(T)$: diffeomorphisms of boundary
= Trajectory of bdry particle



Euclidean $T \to -i\tau$

$$I_{JT} = -\frac{\phi_b}{8\pi G}\int_0^\beta d\ \{t, \tau\}$$ periodic in $\tau \sim \tau+\beta$

Equations of motion obtained by varying $t \to t + \delta t$

use $\int d\tau\, \delta\{t,\tau\} = -\int d\tau\, \frac{\{t,\tau\}'}{t'}\,\delta t$ To integrate by parts

$$\Rightarrow \quad \frac{d}{d\tau}\{t, \tau\} = 0$$: $\{t,\tau\}$ is conserved (by $SL(2,\mathbb{R})$ symmetry)

Solved by $t(\tau) = \tan\frac{\pi\tau}{\beta}$: Thermal AdS_2

The Schwarzian action can be exactly quantized.

2D black holes from Hi-D gravity

martes, 6 de agosto de 2024 18:14

Many lo-D black holes appear in the <u>spherical reduction</u> of a higher D black hole, near the horizon, and often in a low-energy/low-temperature (near-extremal or not) limit.

The simplest is AdS_2 from near-horizon (near-) extremal Reissner-Nordström (in any D)

$$ds^2 = -\frac{f(r)}{\left(1+\frac{q}{r}\right)^2}dt^2 + \left(1+\frac{q}{r}\right)^2\left(\frac{dr^2}{f(r)} + r^2 d\Omega_2\right) \qquad f(r) = 1 - \frac{r_0}{r}$$

Charge $\propto q$ $\qquad r_0$ = non-extremality parameter

$$\boxed{\begin{array}{l}
\text{This is not the usual area-radius gauge:} \\
\tilde{r} = r + q \\
\dfrac{f(r)}{\left(1+\frac{q}{r}\right)^2} = \dfrac{r(r-r_0)}{(r+q)^2} = \dfrac{(\tilde{r}-q)(\tilde{r}-q-r_0)}{\tilde{r}^2} = 1 - \dfrac{2M}{\tilde{r}} + \dfrac{Q^2}{\tilde{r}^2} \\
M = q + r_0/2 \qquad r_+ = M + \sqrt{M^2-Q^2} = r_0 + q \\
Q^2 = q(q+r_0) \qquad M^2 - Q^2 = r_0^2/4
\end{array}}$$

Horizon at $r = r_0$. Area $= 4\pi(r_0+q)^2$

Extremal limit: $r_0 = 0$, horizon at $r=0$, finite area

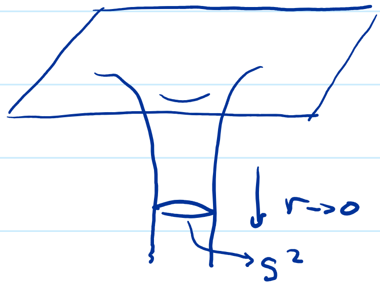
In this case take $r \ll q$, near-horizon limit:

In this case taking $r \ll q$, near-horizon limit:

$$ds^2 \simeq -\frac{r^2}{q^2}dt^2 + q^2\frac{dr^2}{r^2} + q^2 d\Omega_2 \qquad r = \frac{q^2}{z}$$

$$= q^2\left(\frac{-dt^2 + dz^2}{z^2} + d\Omega_2\right)$$

$\underline{\text{Poincaré-AdS}_2} \times S^2$



$$\text{spatial distance} \propto \int \frac{dr}{r} = \log r \xrightarrow{r\to 0} -\infty$$

Now consider near-extremal, near-horizon

Take $r, r_0 \ll q$

$$ds^2 = -\frac{r(r-r_0)}{q^2}dt^2 + q^2\frac{dr^2}{r(r-r_0)} + q^2 d\Omega_2$$

change $r = \frac{r_0}{2}(\tilde{r}+1)$ $\quad t = \frac{2q^2}{r_0}\tilde{t}$. Then

$$ds^2 = q^2\left(-(\tilde{r}^2-1)d\tilde{t}^2 + \frac{d\tilde{r}^2}{\tilde{r}^2-1} + d\Omega_2\right)$$

$\underline{\text{Rindler-AdS}_2} \times S^2$

or $\quad r = r_0\cosh^2\rho/2 \qquad t = \frac{2q^2}{r_0}\tilde{t} \qquad \rho = 2\,\text{arctanh}\sqrt{\frac{r-r_0}{r}}$

$$ds^2 = q^2\left(-\sinh^2\rho\, d\tilde{t}^2 + d\rho^2 + d\Omega_2\right)$$

To get Global $AdS_2 \times S^2$ we should start w/ a wormhole

global AdS_2 is wormhole-like

$-g_{tt} = r^2 + \kappa$ $\quad$ $\kappa = -1$: gravitational attraction: +ve energy

$\kappa = +1$ $\quad$ " $\quad$ repulsion: -ve energy

One can obtain JT gravity as the theory that describes the near-horizon dynamics of near-extremal charged black holes